System Reliability Modeling Considering Correlated Probabilistic Competing Failures

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***Abstract*—A combinatorial system reliability modeling method is proposed to consider the effects of correlated probabilistic competing failures caused by the probabilistic–functional-dependence (PFD) behavior. PFD exists in many real-world systems, such as sensor networks and computer systems, where functions of some system components (referred to as dependent components) rely on functions of other components (referred to as triggers) with certain probabilities. Competitions exist in the time domain between a trigger failure and propagated failures of corresponding dependent components, causing a twofold effect. On one hand, if the trigger failure happens first, an isolation effect can take place preventing the system function from being compromised by further dependent component failures. On the other hand, if any propagated failure of the dependent components happens before the trigger failure, the propagation effect takes place and can cause the entire system to fail. In addition, correlations may exist due to the shared trigger or dependent components, which make system reliability modeling more challenging. This paper models effects of correlated, probabilistic competing failures in reliability analysis of nonrepairable binary-state systems through a combinatorial procedure. The proposed method is demonstrated using a case study of a relay-assisted wireless body area network system in healthcare. Correctness of the method is verified using Monte–Carlo simulations.**

***Index Terms*—Combinatorial method, correlated probabilistic competing failure, failure isolation, failure propagation, probabilistic functional dependence (PFD), wireless body area network (WBAN).**

NOMENCLATURE

BDD	Binary decision diagram.
DFD	Deterministic functional dependence.
FT	Fault tree.
IC	Information category.
IFG	Isolation factor group.
LF	Local failure.
PDEP	Probabilistic dependent.
pdf	Probability density function.
PF	Propagated failure.
PFD	Probabilistic functional dependence.
PFDG	Probabilistic-functional-dependence gate.
PFE	Propagated failure event.
PFGE	Propagated failure with global effect.
PFSE	Propagated failure with selective effect.
SEA	Simple and efficient algorithm.
TLFE	Trigger local failure event.
TPFE	Trigger propagated failure event.
WBAN	Wireless body area network.
AND, OR	Logic operations.
TLFE_j	Trigger local failure event.
$D_{j,k}$	Isolation case under TLFE_j.
e_i	PDEP component.
E_i	Event representing isolation of PDEP component e_i.
$G(\mathrm{TLFE}_j)$	Set of PDEP components affected by active triggers in TLFE_j.
e_{jx}	PDEP component in $G(\mathrm{TLFE}_j)$.
E_{jx}	Event representing isolation of PDEP component e_{jx}.
$f_i(t)$	pdf of time-to-failure of component *i*.
m	Number of trigger components in system.
n_j	Number of PDEP components in $G(\mathrm{TLFE}_j)$.
t	Mission time.
$q_{iC}(t)$	Conditional failure probability of component *i* given that no PFGE occurs to *i* at time *t*.
$q_{iL}(t)$, $q_{iP}(t)$	Unconditional LF, PF probability of component *i* at time *t*.
$q_{T\to E_i}$	Conditional occurrence probability of E_i given the occurrence of trigger failure event *T*.
$q_{\mathrm{TLFE}_j\to E_{jx}}$	Conditional occurrence probability of E_{jx} (i.e., component e_{jx} is isolated) given the occurrence of its corresponding trigger failure event(s) under TLFE_j.
r_i	Relay in the example WBAN.
R_i	Failure event of relay r_i.
R_{iC}	Conditional failure event of r_i given that no PFGE occurs to r_i.
$\bar{R}_{iC}$	Event that r_i functions correctly.
R_{iP}	Event that r_i fails globally.
s_i	Biosensor in the example WBAN.

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S_i	Event representing failure or isolation of biosensor s_i.
S_{iC}	Conditional failure event of s_i given that no PFGE occurs to s_i.
$\bar{S}_{iC}$	Event that s_i functions correctly.
S_{iP}	Event that s_i fails globally.
$S(D_{j,k})$	Set of components involved in competition and isolated by their corresponding trigger failures under $D_{j,k}$.
$\bar{S}(D_{j,k})$	Set of components not involved in competition under $D_{j,k}$.
$q_{R_i \to S_i}$	Conditional occurrence probability of S_i conditioned on the occurrence of relay failure event R_i.
t_i	Trigger component.
T_i	Event representing failure of trigger component t_i.
$U_{\text{sys}}(t)$	System failure probability at time *t*.
$X \to Y$	Event that *X* happens before *Y*.
$\alpha,\ \beta$	Shape, scale parameter of Weibull distribution.

I. Introduction

Many real-world systems are subject to probabilistic-functional-dependence (PFD) behavior, where operations of some system components are probabilistically dependent on functions of other components within the same system. For example, in a relay-assisted wireless body area network (WBAN), a biosensor can deliver physiological information to the sink device through a relay node in order to reduce energy consumption and mitigate path loss [1]. When the relay node fails, the biosensor may increase its transmission power to be wirelessly and directly connected to the sink device with a certain probability related to the percentage of remaining power. In other words, the biosensor becomes inaccessible or isolated to the WBAN if its remaining energy is inadequate to support the direct transmission to the sink device at the time of the corresponding relay failure. In this case, we say that the biosensor has the PFD on the relay. The relay and its corresponding biosensors form a PFD group, where the relay is referred to as a trigger and the biosensors are referred to as probabilistic-dependent (PDEP) components.

A component can experience a local failure (LF) that only disables the single component itself [2], e.g., hard disk or memory unit failure in a computer. It can also be subject to a propagated failure (PF) that not only incapacitates the component itself but also affects other system components [3]. Propagated failure with global effect (PFGE) and selective effect (PFSE) can be distinguished according to the scope of the damage caused by the failure. Specifically, a PFSE compromises only a subset of system components when occurring; refer to [3] for the study of PFSE. As one type of common-cause failures, a PFGE originating from any component can lead to the entire system failure. It can generally be caused by imperfect fault coverage, destructive effect, or malicious attacks. For example, an undetected component fault (due to a recovery mechanism failure) could propagate throughout the system; or some component failure can cause overheating, fire, short circuit, etc. PFGE is assumed in this study. For the WBAN example, each component (biosensor or relay) can be subject to a LF due to disabled transmission function and a PFGE due to jamming attacks, which are launched by continually transmitting interference signals from a compromised component to the sink device.

Competitions exist in the time domain between a trigger LF and PFGE of corresponding PDEP components within the same PFD group, causing a twofold effect. On one hand, if the trigger failure occurs first, an isolation effect can take place preventing the system function from being compromised by further PFGE originating from the PDEP component. On the other hand, if any PFGE of the PDEP components occurs before the trigger failure, the propagation effect takes place causing the entire system to fail. Consider the WBAN example again. The above described probabilistic competition exists between the LF of a relay (i.e., probabilistic isolation effect) and the PFGE of biosensors (i.e., failure propagation effect) within the same PFD group. If the relay LF occurs first, a biosensor can be isolated from the WBAN with a certain probability related to its remaining power and such an isolation effect prevents PFGE of the isolated biosensor from affecting the rest of WBAN. However, if a PFGE originating from a biosensor happens first, the failure propagation can compromise the entire WBAN. In addition, a biosensor may deliver its sensed information to the sink device through multiple relays and a relay may be shared by multiple biosensors. Such dependencies among different relay-biosensor groups (i.e., PFD groups) make the probabilistic competing failure analysis more challenging.

This paper makes new contributions by addressing correlated probabilistic competing failures in reliability analysis of nonrepairable binary-state systems. A combinatorial method is proposed, which has no limitation on the types of time-to-LF or PFGE distributions as well as distributions modeling the probabilistic isolation effect.

The remainder of this paper is organized as follows: Section II reviews related research on competing failures. Section III presents the PFD behavior modeling and a preliminary method for handling PFGEs in system reliability evaluation. Section IV presents the proposed combinatorial method. Section V illustrates application and advantages of the proposed method through a case study of a WBAN system subject to correlated probabilistic competing failures. Analysis results and verification of the proposed method using Monte–Carlo simulations are also presented. Finally, Section VI gives conclusions as well as directions for future work.

II. Related Research

Considerable research efforts were dedicated to consider competing failures in system reliability analysis. For example, system reliability modeling considering multiple competing failure causes (e.g., degradation, catastrophic failures) was investigated for both *s*-independent [4], [5] and *s*-dependent [6], [7] scenarios. Competing processes of multiple failure modes or causes were also investigated in the context of accelerated life tests [8]–[10] and system maintenance policy designs [11]–[14]. Particularly in [13], random usage that increases the amount of degradation measure was considered in reliability modeling and maintenance design for a competing risk system. In [14], taking into account dependencies among different degradation processes within one component and of different components, the criticality of components was investigated in condition-based preventive maintenance and corrective maintenance. The system size optimization problem considering competing open and short failure modes was addressed for *k*-out-of-*n* systems [15] and parallel–series systems [14]. In [17], the lifetime distribution and limiting availability were investigated for a periodically inspected single-unit system subject to competing degradation wear (characterized by a continuous Markov chain) and random shocks (modeled by a homogeneous Poisson process). In [18], reliability and availability were modeled for systems subject to several stage-dependent degradations with partial repairs.

In addition, competing failures were investigated in the context of imperfect fault coverage [19]–[24], where uncovered (or propagated) and covered component fault modes are differentiated and modeled. Competing failures addressed in all of the previously mentioned work were mainly concerned with competitions among multiple types of failure causes or modes; whereas the competing failures considered in this paper are concerned with competitions between global failure propagation and probabilistic failure isolation caused by the PFD behavior.

Recent work [1], [25]–[29] has investigated reliability modeling of systems subject to deterministic functional dependence (DFD), where the occurrence of a trigger LF (if it happens first) causes a *deterministic* or *certain* isolation effect (i.e., with occurrence probability of 1) to the corresponding dependent components. Motivated by work in [1] and [25], a selective maintenance policy was proposed for systems displaying the DFD behavior [30]. However, the models developed for addressing the DFD behavior are not applicable in addressing the general PFD behavior considered in this paper, where the occurrence of a trigger LF (if it happens first) causes a *probabilistic* or *uncertain* isolation effect to its corresponding PDEP components with different occurrence probabilities (typically less than 1). In our preliminary work [31], the probabilistic competing failures were studied in a body sensor network system with only one relay-biosensor or PFD group. However, multiple-dependent PFD groups sharing a common trigger or PDEP component can exist in real-world systems [1]. Reliability modeling of such systems requires a new solution method that is able to accommodate the correlated probabilistic failure competition. We make the original contribution by suggesting a combinatorial procedure of analyzing reliability of nonrepairable binary systems subject to multiple dependent PFD groups.

III. Preliminary Models

This section presents the modeling of PFD, and an analytical method for considering PFGEs in system reliability evaluation.

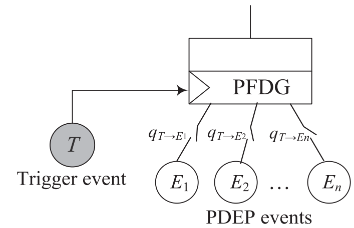

Fig. 1. General structure of a PFDG.

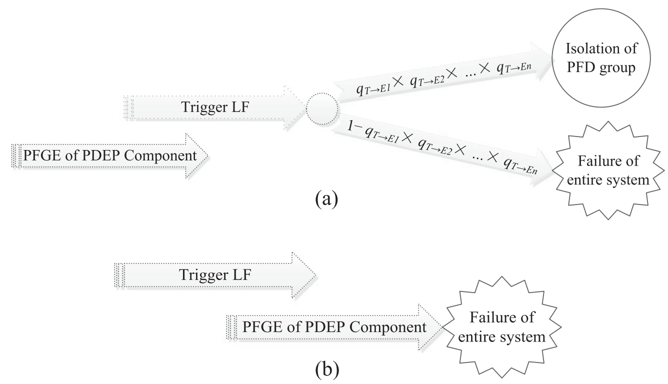

Fig. 2. Probabilistic competing failures. (a) Probabilistic isolation effect. (b) Failure propagation effect.

A. PFD Modeling

Failure behaviors of systems investigated in this work are modeled using fault trees (FT) [22], [32], [33]. Traditional FT models, however, cannot represent the PFD behavior. We introduce a new gate called probabilistic-functional–dependence gate (PFDG) to model this dynamic behavior. Fig. 1 illustrates the general structure of the PFDG gate, which is composed of a single trigger event and one or more PDEP events. Note that the trigger event can be a single trigger component LF or a combination of multiple trigger component LFs. The PDEP events are referred to as events that the PDEP components become isolated (inaccessible or unusable). The occurrence of the trigger event causes the corresponding PDEP events E_i to occur with different conditional probabilities $q_{T\to Ei}$ $(i = 1, 2, \ldots, n)$, which is modeled by the *switch symbols* in the PFDG. In other words, the failure of trigger components can probabilistically isolate the corresponding PDEP components.

As mentioned in Section I, competitions exist in the time domain between a trigger LF and PFGE from PDEP components in the same PFD group, and different occurrence orderings can lead to different system statuses. As illustrated in Fig. 2(a), the trigger LF (if it happens first) causes a probabilistic isolation effect that can isolate the corresponding PDEP components and their PFGE with probability $q_{T\to E_1} \times q_{T\to E_2} \times \ldots \times q_{T\to E_n}$. Under the isolation effect, all the PDEP components are inaccessible or unusable, and, thus, are considered as being failed for system reliability analysis. However, if a PFGE originating from any PDEP components is not isolated (see Fig. 2(a) with probability $1 - q_{T\to E_1} \times q_{T\to E_2} \times \ldots \times q_{T\to E_n}$) or occurs before the trigger LF (see Fig. 2(b)), the entire system failure happens with a probability of 1 due to the global failure propagation effect from the nonisolated PFGE.

B. Separable Method for Addressing PFGEs

To address PFGEs in system reliability evaluation, a simple and efficient algorithm (SEA) was proposed [20], [21], where two complementary propagated failure events (PFEs) are differentiated.

1) PFE_1—At least one PFGE originating from a system component occurs by time *t*.
2) PFE_2—No PFGE occurs to the system by time *t*.

Two conditional system failure events can be defined correspondingly, and their occurrence probabilities can be aggregated based on the total probability law as

$$U_{\mathrm{sys}}(t) = \Pr(\text{system fails at } t|\mathrm{PFE}_1) \times \Pr(\mathrm{PFE}_1) + \Pr(\text{system fails at } t|\mathrm{PFE}_2) \times \Pr(\mathrm{PFE}_2) \quad (1)$$

where $\Pr(\mathrm{PFE}_1)$ is simply equal to $1 - \Pr(\mathrm{PFE}_2)$, and $\Pr(\text{system fails at } t|\mathrm{PFE}_1) = 1$. Effects of all the PFGEs are actually separated from the evaluation of $\Pr(\text{system fails at } t|\mathrm{PFE}_2)$, which can, thus, be solved using any traditional reliability analysis approach ignoring PFGEs, e.g., binary decision diagram (BDD)-based methods [32], [34]. The SEA is incorporated into our proposed method to handle the failure propagation effects of system components in Section IV.

IV. Proposed Combinatorial Method

This section presents a combinatorial methodology for reliability analysis of nonrepairable binary-state systems having multiple correlated PFD groups. The proposed method is presented in Section IV–A. Its complexity is discussed in Section IV–B. A case study illustrating the proposed method is given in Section V.

A. Combinatorial Method

The proposed method addresses correlated, probabilistic competing failures and is described by the following four-step process.

Step 1: Separate PFGEs of Triggers

PFGEs originating from trigger components can bring down the entire system regardless of the states of other system components. Their effects can be separated from the overall solution combinatorics using the idea of the SEA method (see Section III–B). Specifically, two complementary PFEs of triggers, referred to as TPFEs are distinguished.

1) TPFE_1—At least one PFGE originating from a trigger component occurs by time *t*.
2) TPFE_2—No PFGE occurs to any trigger component by time *t*.

Applying (1), we have

$$U_{\mathrm{sys}}(t) = \Pr(\text{system fails at } t|\mathrm{TPFE}_1) \times \Pr(\mathrm{TPFE}_1) + \Pr(\text{system fails at } t|\mathrm{TPFE}_2) \times \Pr(\mathrm{TPFE}_2). \quad (2)$$

$\Pr(\mathrm{TPFE}_1)$ simply equals to $1 - \Pr(\mathrm{TPFE}_2)$. $\Pr(\text{system fails at } t|\mathrm{TPFE}_1)$ is 1 since a PFGE from any of the triggers can crash the entire system. Thus, $U_{\mathrm{sys}}(t)$ can be rewritten as

$$U_{\mathrm{sys}}(t) = 1 - \Pr(\mathrm{TPFE}_2) + \Pr(\text{system fails at } t|\mathrm{TPFE}_2) \times \Pr(\mathrm{TPFE}_2). \quad (3)$$

Define *m* as the number of triggers and the probability of trigger t_i experiencing a PFGE is $q_{t_i P}(t)$, we have

$$\Pr(\mathrm{TPFE}_2) = \prod_{i=1}^{m} \Pr(\text{no PFGE occurs to } t_i) = \prod_{i=1}^{m} (1 - q_{t_i P}(t)). \quad (4)$$

The method of evaluating $\Pr(\text{system fails at } t|\mathrm{TPFE}_2)$ in (3) is described in the following steps. To simplify the representation, we omit the mission time *t* in the subsequent equations and discussions.

Step 2: Construct Trigger Local Failure Events (TLFEs)

Since PFGEs of triggers have been separated in Step 1, only LFs of triggers are considered in this step. Based on the working or locally failed states of the *m* triggers, an event space that consists of 2^m combined TLFEs can be constructed. Thus, the size of the TLFE space is exponential to the number of trigger components in the system. Each TLFE_j $(j = 1, 2, \ldots, 2^m)$ is a combination of occurrence or nonoccurrence of LFs of the *m* trigger components t_i as follows:

$$\begin{aligned} \mathrm{TLFE}_1 &= \overline{T}_1 \cap \overline{T}_2 \cdots \cap \overline{T}_m \\ \mathrm{TLFE}_2 &= T_1 \cap \overline{T}_2 \cdots \cap \overline{T}_m \\ &\;\;\cdots \\ \mathrm{TLFE}_{2^m} &= T_1 \cap T_2 \cdots \cap T_m. \end{aligned} \quad (5)$$

Each TLFE_j can cause probabilistic failure isolation effects to a different subset of system components. Related PDEP components affected by active trigger event T_i appear in the definition of TLFE_j, which form a set denoted as $G(\mathrm{TLFE}_j)$. For example, $G(\mathrm{TLFE}_1)$ is simply *NULL* as none of trigger events is active; $G(\mathrm{TLFE}_2)$ is the set of PDEP components within the PFD group of T_1 since only T_1 is active under TLFE_2; $G(\mathrm{TLFE}_2^m)$ is the union of sets of PDEP components within the PFD groups of all triggers since all the *m* trigger events are active.

Based on the total probability law, $\Pr(\text{system fails}|\mathrm{TPFE}_2)$ is computed as

$$\Pr(\text{system fails}|\mathrm{TPFE}_2) = \sum_{j=1}^{2^m} [\Pr(\text{system fails}|\mathrm{TLFE}_j) \times \Pr(\mathrm{TLFE}_j)]. \quad (6)$$

Methods of evaluating $\Pr(\mathrm{TLFE}_j)$ and $\Pr(\text{system fails}|\mathrm{TLFE}_j)$ are presented in Step 3.

Step 3: Probabilistic Competing Failure Analysis for TLFE_j

By definition of each TLFE_j, $\Pr(\mathrm{TLFE}_j)$ can be computed via simply multiplying occurrence or nonoccurrence probabilities of trigger LFs.

To evaluate $\Pr(\text{system fails}|\mathrm{TLFE}_j)$, the following two scenarios are considered differently.

1) In the event of TLFE_1 defined in (5) occurring, all trigger components function correctly, so neither failure competition nor probabilistic failure isolation exists. To evaluate $\Pr(\text{system fails}|\mathrm{TLFE}_1)$, the original system FT model can be reduced by removing all the trigger events and all the PFDG gates. Based on the reduced FT, $\Pr(\text{system fails}|\mathrm{TLFE}_1)$ can be evaluated by applying the SEA method of Section III–B.
2) In the event of TLFE_j $(j > 1)$ occurring, different isolation cases are first identified. Each case corresponds to a different combination of isolation of PDEP components by the corresponding triggers associated with TLFE_j. Define n_j as the number of PDEP components in $G(\mathrm{TLFE}_j)$, which are denoted by e_{jx}, $x = 1, 2, \ldots, n_j$ with their isolation events denoted by E_{jx}. 2^{n_j} isolation cases that further decompose TLFE_j can then be identified as follows, each denoted as $D_{j,k} (k = 1, 2, \ldots, 2^{n_j})$

$$\begin{aligned}
D_{j,1} &= \overline{E}_{j1} \cap \overline{E}_{j2} \cdots \cap \overline{E}_{jn_j}\\
D_{j,2} &= E_{j1} \cap \overline{E}_{j2} \cdots \cap \overline{E}_{jn_j}\\
&\cdots\\
D_{j,2^{n_j}} &= E_{j1} \cap E_{j2} \cdots \cap E_{jn_j}.
\end{aligned} \tag{7}$$

For example, $D_{j,1}$ means no PDEP component is actually isolated; $D_{j,2}$ means only $e_{j,1}$ is isolated by its corresponding trigger LF; $D_{j,2^{n_j}}$ means all components of $G(\mathrm{TLFE}_j)$ are isolated by their corresponding trigger LFs. The occurrence probability of $D_{j,k}$ is computed using parameters modeling probabilistic isolation relationships between the trigger failure event and its corresponding isolation events of PDEP components, i.e., $q_{T\to Ei}$ introduced in Section III–A. For example,

$$\begin{aligned}
\Pr(D_{j,1}) &= \left(1 - q_{\mathrm{TLFE}_j\to E_{j1}}\right) \times \left(1 - q_{\mathrm{TLFE}_j\to E_{j2}}\right)\\
&\quad \times \cdots \times \left(1 - q_{\mathrm{TLFE}_j\to E_{jn_j}}\right)\\
\Pr(D_{j,2}) &= q_{\mathrm{TLFE}_j\to E_{j1}} \times \left(1 - q_{\mathrm{TLFE}_j\to E_{j2}}\right)\\
&\quad \times \cdots \times \left(1 - q_{\mathrm{TLFE}_j\to E_{jn_j}}\right)\\
&\cdots\\
\Pr\left(D_{j,2^{n_j}}\right) &= q_{\mathrm{TLFE}_j\to E_{j1}} \times q_{\mathrm{TLFE}_j\to E_{j2}}\\
&\quad \times \cdots \times q_{\mathrm{TLFE}_j\to E_{jn_j}}.
\end{aligned} \tag{8}$$

Note that in $G(\mathrm{TLFE}_j)$, each PDEP component may be isolated by different specific failed trigger(s), and, thus, the general notation $q_{\mathrm{TLFE}_j\to E_{jx}}, x = 1, 2, \ldots, n_j$ is used in (8) to represent the probability that E_{jx} happens given the occurrence of its corresponding trigger failure event(s).

The set of isolated components associated with each $D_{j,k}$, denoted by $S(D_{j,k})$ is a subset of $G(\mathrm{TLFE}_j)$. For example, $S(D_{j,1})$ is *NULL* since no component is isolated by the failed trigger under $D_{j,1}$; $S(D_{j,2}) = \{e_{j,1}\}$; and $S(D_{j,2^{n_j}}) = G(\mathrm{TLFE}_j)$. Components not belonging to $S(D_{j,k})$ form a set denoted as $\bar{S}(D_{j,k})$, and they are not involved in the competition. At any time, a PFGE originating from any components in $\bar{S}(D_{j,k})$ causes the entire system failure.

The time-domain competition exists between LFs of triggers and PFGEs from components in $S(D_{j,k})$. As depicted in Fig. 2, the competition results in twofold effects: the failure propagation effect that causes the entire system to fail; and the failure isolation effect that causes a particular PFD group to be isolated (the system state depends on states of the remaining components and system structure function or configuration). Similar to the SEA method stated in Section III–B, for each TLFE_j, two PFEs can be differentiated to address the twofold effects.

1) $\mathrm{PFE}_{j,1}$—At least one PFGE originating from a PDEP component occurs.
2) $\mathrm{PFE}_{j,2}$—No failure propagation occurs to the system.

In terms of the isolation cases $D_{j,k}$ $(k = 1, 2, \ldots, 2^{n_j})$ identified in TLFE_j, two disjoint events are further distinguished for each $D_{j,k}$:

$D_{j,k} \cap \mathrm{PFE}_{j,1}$: This event happens when at least one PFGE originating from components in $S(D_{j,k})$ is not isolated (i.e., the corresponding trigger does not fail or fails after the occurrence of PFGE), or any PFGE from components in $\bar{S}(D_{j,k})$ takes place. This event causes the failure propagation effect and its occurrence probability $\Pr(D_{j,k} \cap \mathrm{PFE}_{j,1})$ can be evaluated in a straightforward way as illustrated in Section V.

$D_{j,k} \cap \mathrm{PFE}_{j,2}$: This event happens when all PFGEs from components in $S(D_{j,k})$ are isolated by corresponding trigger LFs (i.e., occur after the corresponding triggers fail) and no PFGE from components in $\bar{S}(D_{j,k})$ occurs; or no PFGE occurs to any system components during the mission time. For evaluating $\Pr(D_{j,k} \cap \mathrm{PFE}_{j,2})$, all combinations of occurrence or nonoccurrence of all PFGEs from components in $S(D_{j,k})$ should be considered, which is much more complicated than the evaluation of $\Pr(D_{j,k} \cap \mathrm{PFE}_{j,1})$. Therefore, in this step, the occurrence probability $\Pr(D_{j,k} \cap \mathrm{PFE}_{j,1})$ is computed first, and $\Pr(D_{j,k} \cap \mathrm{PFE}_{j,2})$ can be obtained as

$$\begin{aligned}
\Pr(D_{j,k} \cap \mathrm{PFE}_{j,2}) = {} & \Pr(\mathrm{TLFE}_j) \times \Pr(D_{j,k})\\
& - \Pr(D_{j,k} \cap \mathrm{PFE}_{j,1}).
\end{aligned} \tag{9}$$

Based on the 2^{n_j+1} combined events that decompose TLFE_j ($D_{j,k} \cap \mathrm{PFE}_{j,1}$ and $D_{j,k} \cap \mathrm{PFE}_{j,2}$ for $k = 1, 2, \ldots, 2^{n_j}$) and total probability law, we have

$$\begin{aligned}
&\Pr(\text{system fails}|\mathrm{TLFE}_j) \times \Pr(\mathrm{TLFE}_j)\\
&= \sum_{k=1}^{2^{n_j}} [\Pr(\text{system fails}|D_{j,k} \cap \mathrm{PFE}_{j,1}) \times \Pr(D_{j,k} \cap \mathrm{PFE}_{j,1})\\
&\qquad + \Pr(\text{system fails}|D_{j,k} \cap \mathrm{PFE}_{j,2})\\
&\qquad \times \Pr(D_{j,k} \cap \mathrm{PFE}_{j,2})]\\
&= \sum_{k=1}^{2^{n_j}} [\Pr(D_{j,k} \cap \mathrm{PFE}_{j,1}) + \Pr(\text{system fails}|D_{j,k} \cap \mathrm{PFE}_{j,2})\\
&\qquad \times \Pr(D_{j,k} \cap \mathrm{PFE}_{j,2})].
\end{aligned} \tag{10}$$

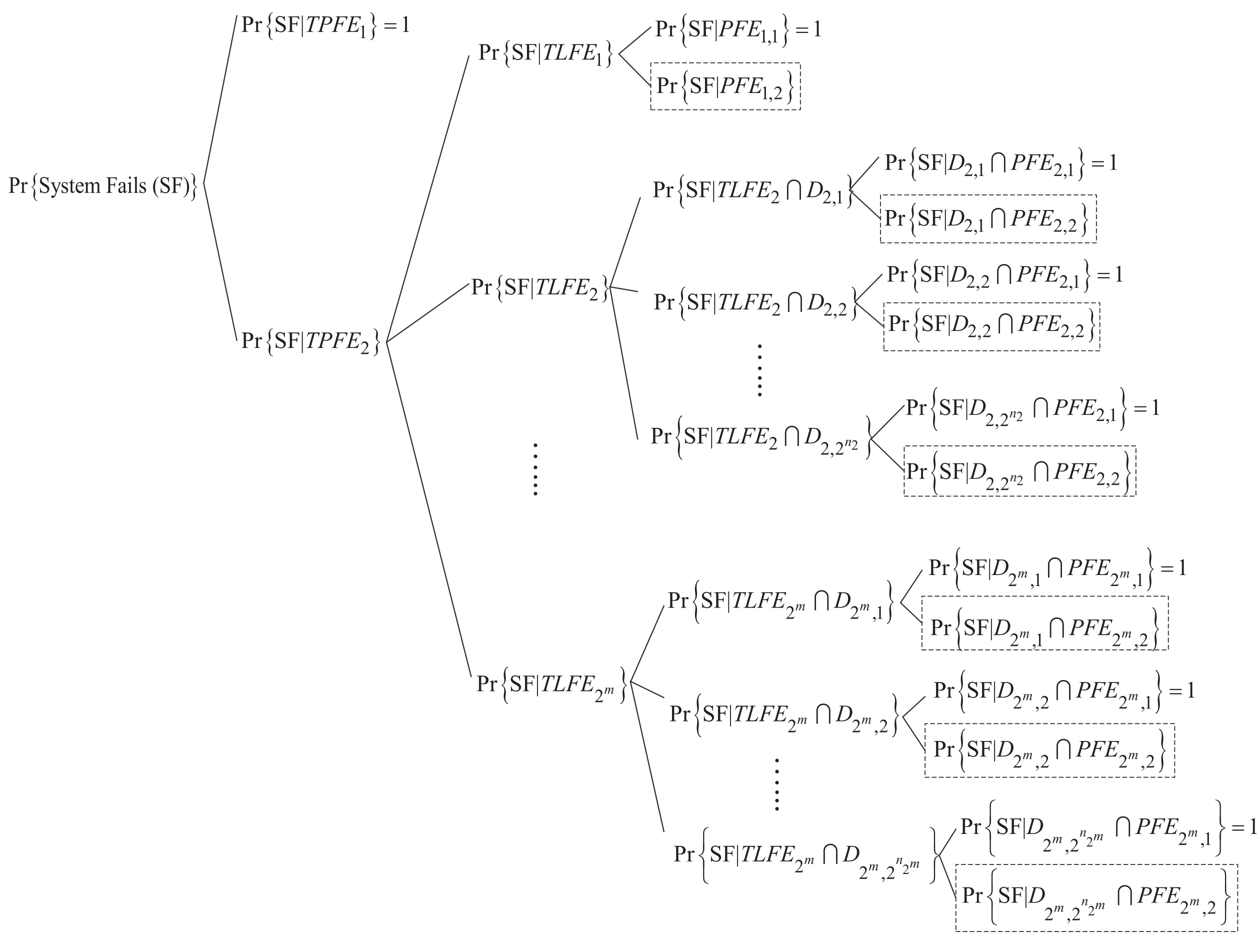

Fig. 3. Problem decomposition tree.

In (10) $\Pr(\text{system fails}|D_{j,k} \cap \text{PFE}_{j,1}) = 1$ since when event $D_{j,k} \cap \text{PFE}_{j,1}$ happens, the failure propagation effect occurs causing the entire system failure. When event $D_{j,k} \cap \text{PFE}_{j,2}$ happens, the failure isolation effect takes place, $\Pr(\text{system fails}|D_{j,k} \cap \text{PFE}_{j,2})$ can be obtained by generating and evaluating a reduced FT, where the triggers and corresponding PFDG gates are removed and the components in $S(D_{j,k})$ are replaced with constant "1" (TRUE).

Step 3 is repeated until all TLFEs are analyzed.

Step 4: Integration for System Unreliability

Based on results obtained in Step 3, $\Pr(\text{system fails}|\text{TPFE}_2)$ can be evaluated using (6). The final system unreliability considering correlated probabilistic competing failures is calculated by (3).

B. Complexity Analysis

As explained previously in the four-step procedure, the proposed method decomposes the original reliability problem into a set of reduced problems. Fig. 3 illustrates the problem decomposition tree of the proposed method.

For systems with multiple correlated PFD groups considered in this study, the size of TPFE space is constant 2. The size of TLFE space is exponential to the number (m) of trigger components. For TLFE_j in scenario 1 ($j = 1$), the PFE space size is constant 2. For each TLFE_j in scenario 2 ($j > 1$), in the case of n_j PDEP components being involved in the PFD behavior, the isolation case ($D_{j,k}$) space is 2^{n_j}, and the combined event ($D_{j,k} \cap \text{PFE}_{j,1}$ and $D_{j,k} \cap \text{PFE}_{j,2}$) space is increased to 2^{n_j+1}. Among the 2^{n_j+1} combined events, 2^{n_j} of them, when occurring, can cause the entire system to fail (i.e., system failure probability of 1 given their occurrences), and, thus, the complexity space of TLFE_j ($j > 1$) is reduced to 2^{n_j}, which is exponential to the number of PDEP components. Therefore, the complexity of reduced problem space is $O(2^{n_j} \times (2^m - 1))$. All the reduced problems are independent, and, thus, can be solved in parallel given adequate computing resources.

V. Illustrative Example

A case study of an example WBAN system for patient monitoring is performed to illustrate the proposed method in this section. Section V–A describes the example WBAN system subject to the PFD behavior. Section V–B analyzes reliability of the example WBAN system using the proposed combinatorial method. Section V–C presents evaluation results and discussion of the results. Verification of the method using Monte–Carlo simulation is presented in Section V–D.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

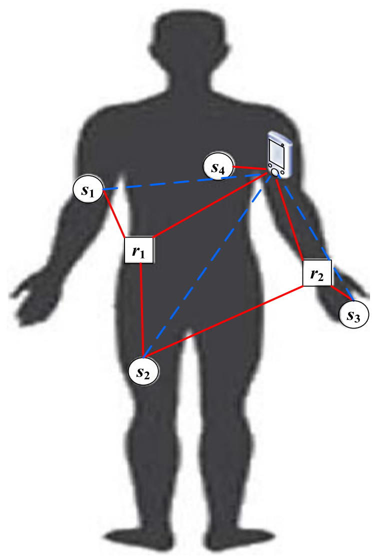

Fig. 4. Example WBAN system for healthcare.

A. System Description

As shown in Fig. 4, the example WBAN system consists of four biosensors (s_1, s_2, s_3, and s_4) and two relays (r_1 and r_2). The biosensors are used to measure essential physiological information, such as blood pressure, EMG, SpO_2, and heart rates. To improve the robustness and reduce energy consumption, biosensors s_1 and s_3 can deliver sensed information through relay r_1 and r_2, respectively; s_2 can deliver acquired data via either r_1 or r_2. s_4 can transmit the information directly to the sink device. The sink device gathers information from the WBAN system and communicates with physicians via an external gateway. The sink device is assumed to be perfectly reliable. Both biosensors and relays can experience LFs due to transmission malfunctions and PFGEs due to jamming attacks. Since the jamming attacks are launched by continually transmitting interference signals to the sink device, the LF and PFGE of the same WBAN component are mutually exclusive.

Let $q_{iL}(t)$ and $q_{iP}(t)$ be unconditional LF and PFGE probabilities of component *i*, respectively. Given the mutually exclusive relationship between LF and PFGE, the conditional failure probability given that no PFGE occurs to component *i*, denoted by $q_{iC}(t)$, can be calculated as

$$\begin{aligned} q_{iC}(t) &= \Pr(\text{LF}|\text{noPFGE}) \\ &= \frac{\Pr(\text{LF}) \times \Pr(\text{no PFGE}|\text{LF})}{\Pr(\text{no PFGE})} \\ &= \frac{\Pr(\text{LF})}{\Pr(\text{no PFGE})} \\ &= \frac{q_{iL}(t)}{1 - q_{iP}(t)}. \end{aligned} \tag{11}$$

In the example WBAN, when relays r_1 and r_2 fail locally, biosensors s_1, s_2, and s_3 need adequate remaining energy to be wirelessly and directly connected to the sink device by increasing transmission power; otherwise, these biosensors are isolated from the rest of WBAN. Therefore, biosensors s_1, s_2, and s_3 have PFD on relays and can be isolated by the LF of relays with some probability related to the percentage of their remaining energy.

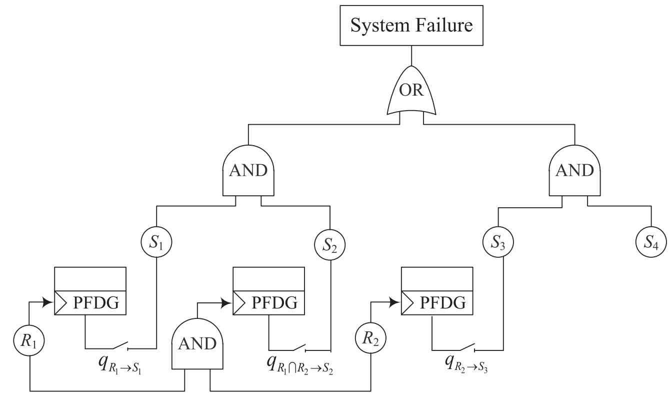

Fig. 5. FT model of the example WBAN system.

TABLE I
PROBABILISTIC ISOLATION EFFECTS AND PROBABILITIES

Trigger Events	S_1	S_2	S_3	S_4
$\bar{R}_1 \cap \bar{R}_2$	0	0	0	N/A
$R_1 \cap \bar{R}_2$	$q_{R1 \to S1}$	0	0	N/A
$\bar{R}_1 \cap R_2$	0	0	$q_{R2 \to S3}$	N/A
$R_1 \cap R_2$	$q_{R1 \to S1}$	$q_{R1 \cap R2 \to S2}$	$q_{R2 \to S3}$	N/A

Fig. 5 shows the FT model of the example WBAN system. For a successful diagnosis, two categories of information (denoted as IC_A and IC_B) are required: IC_A can be provided by either s_1 or s_2; and IC_B can be provided by either s_3 or s_4. In other words, when both s_1 and s_2 fail, IC_A is not available, which is modeled by the left AND gate under the top OR gate; similarly, when both s_3 and s_4 fail, IC_B is not available, which is modeled by the right AND gate under the top OR gate. Three PFDG gates are applied to model the correlated PFD behavior existing between relays and biosensors, where the trigger input event represents the occurrence of relay LF(s), and each PDEP event represents a biosensor being inaccessible to the rest of the WBAN system. Each PFDG gate corresponds to a PFD group; there are three PFD groups for the example WBAN system: $\{r_1, s_1\}$, $\{r_1, r_2, s_2\}$, and $\{r_2, s_3\}$. Given that a relay LF occurs first, biosensors from the same PFD group are isolated from the WBAN with different conditional probabilities, referred to as probabilistic isolation factors. Particularly, $q_{R1 \to S1}$, $q_{R1 \cap R2 \to S2}$, and $q_{R2 \to S3}$ are conditional probabilities that biosensors s_1, s_2, and s_3 can be isolated (i.e., isolation events S_1, S_2, and S_3 can happen), conditioned on the occurrence of their corresponding trigger events, respectively.

Table I summarizes the probabilistic isolation effect on each biosensor under different combinations of two relay LFs. Specifically, if both r_1 and r_2 operate correctly (denoted by "$\bar{R}_1 \cap \bar{R}_2$" in the first column of Table I), no isolation happens to biosensors s_1, s_2, and s_3, and, thus, the probability that these biosensors can be isolated is 0. Since s_4 does not belong to any PFD group, no isolation effect is applicable to this sensor (represented by N/A in the fifth column of Table I). If r_1 fails locally while r_2 operates correctly (denoted by "$R_1 \cap \bar{R}_2$"), only biosen-

TABLE II
EVENT SPACE CONSIDERING LFs OF RELAYS

TLFE_j	Event definition	$G(\mathrm{TLFE}_j)$
TLFE_1	$\bar{R}_{1C} \cap \bar{R}_{2C}$	*NULL*
TLFE_2	$R_{1C} \cap \bar{R}_{2C}$	$\{s_1\}$
TLFE_3	$\bar{R}_{1C} \cap R_{2C}$	$\{s_3\}$
TLFE_4	$R_{1C} \cap R_{2C}$	$\{s_1, s_2, s_3\}$

sor s_1 is probabilistically isolated with probability $q_{R1\rightarrow S1}$. Similarly, if r_1 operates correctly while r_2 fails locally (denoted by "$\bar{R}_1 \cap R_2$"), only biosensor s_3 is probabilistically isolated with probability $q_{R2\rightarrow S3}$. If both r_1 and r_2 fail locally (denoted by "$R_1 \cap R_2$"), biosensors s_1, s_2, and s_3 can be isolated with different probabilities listed in the fifth row of Table I.

This paper focuses only on system-level reliability modeling and evaluation with the assumption that component-level failure parameters and probabilistic isolation factors are known input parameters. In practice, estimation approaches based on collected failure data (e.g., statistical inference [35], Bayesian estimation [36]) are often applied to estimate component failure time distribution functions and related parameters. The probabilistic isolation factors are related to the power consumption model of biosensors and the maximum transmission range of the biosensors when the corresponding relay failure event occurs, which can be estimated by applying the time-dependent link failure model in [37].

B. Reliability Analysis Illustration

This section presents the step-by-step reliability evaluation of the example WBAN system using the proposed methodology.

Step 1: Separate PFGEs of Triggers

Triggers in the example WBAN system involve two relays r_1 and r_2 that can experience PFGEs. Thus, the following two complementary TPFEs can be distinguished:

1) TPFE_1—At least one PFGE from a relay occurs.
2) TPFE_2—No PFGE occurs to any of the relays.

The WBAN system unreliability can be evaluated using (3), where $\Pr(\mathrm{TPFE}_2)$ is calculated by applying (4) as

$$\Pr(\mathrm{TPFE}_2) = [1 - q_{r_1P}(t)][1 - q_{r_2P}(t)]. \tag{12}$$

$\Pr(\text{system fails}|\mathrm{TPFE}_2)$ in (3) is analyzed through the following Step 2 and Step 3.

Step 2: Construct TLFEs

Based on the working or locally failed states of the two relays, $2^2 = 4$ TLFEs are constructed as shown in Table II. Note that R_{iC} represents the conditional failure event of r_i given that no PFGE occurs to r_i, and $\bar{R}_{iC}$ represents the event that r_i functions correctly. Each TLFE_j can cause probabilistic failure isolation effect to a specific subset of biosensors, denoted by $G(\mathrm{TLFE}_j)$ in Table II.

In this step, $\Pr(\text{system fails}|\mathrm{TPFE}_2)$ is computed by applying (6) as

$$\Pr(\text{system fails}|\mathrm{TPFE}_2) = \sum_{j=1}^{4} [\Pr(\text{system fails}|\mathrm{TLFE}_j) \times \Pr(\mathrm{TLFE}_j)]. \tag{13}$$

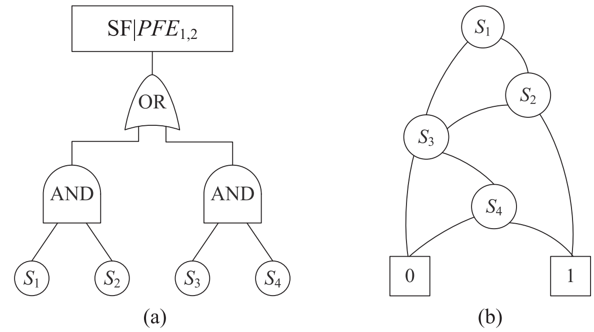

Fig. 6. Reduced system models for TLFE_1. (a) Reduced FT model. (b) BDD model.

Step 3: Probabilistic Competing Failure Analysis for TLFE_j

In this step, $\Pr(\mathrm{TLFE}_j)$ and $\Pr(\text{system fails}|\mathrm{TLFE}_j)$ are evaluated for all TLFEs. Note that the conditional component failure probability $q_{iC}(t)$ evaluated using (11) should be used once the PFGE of component *i* is separated.

1) TLFE_1: Both relays function correctly. The occurrence probability of TLFE_1 is

$$\begin{aligned}\Pr(\mathrm{TLFE}_1) &= \Pr\left(\bar{R}_{1C} \cap \bar{R}_{2C}\right)\\ &= [1 - q_{r_1C}(t)][1 - q_{r_2C}(t)].\end{aligned} \tag{14}$$

In the event of TLFE_1 occurring, both relays are functioning correctly, so neither failure competition nor probabilistic isolation exists. According to scenario 1 of Step 3 in Section IV, to evaluate $\Pr(\text{system fails}|\mathrm{TLFE}_1)$, the system FT model is reduced by removing the two relay events and all the PFDG gates. Based on the reduced model as shown in Fig. 6(a), $\Pr(\text{system fails}|\mathrm{TLFE}_1)$ can be evaluated by applying the SEA method of Section III–B, where PFGEs of the four biosensors are modeled.

Specifically, the following two complementary PFEs that decompose TLFE_1 are identified.

$\mathrm{PFE}_{1,1}$: At least one PFGE from biosensors s_1, s_2, s_3, s_4 occurs crashing the entire system. Thus, $\Pr(\text{system fails}\,|\,\mathrm{PFE}_{1,1}) = 1$ and

$$\begin{aligned}\Pr(\mathrm{PFE}_{1,1}) &= \Pr\left[\left(\bar{R}_{1C} \cap \bar{R}_{2C}\right)\right.\\ &\quad \left.\cap\left(S_{1P} \cup S_{2P} \cup S_{3P} \cup S_{4P}\right)\right].\end{aligned} \tag{15}$$

$\mathrm{PFE}_{1,2}$: No PFGE occurs to the four biosensors. The occurrence probability of $\mathrm{PFE}_{1,2}$ is

$$\Pr(\mathrm{PFE}_{1,2}) = \Pr(\mathrm{TLFE}_1) - \Pr(\mathrm{PFE}_{1,1}). \tag{16}$$

For evaluating $\Pr(\text{system fails}|\mathrm{PFE}_{1,2})$, the BDD method is applied to solve the reduced FT in Fig. 6(a). The BDD model constructed is shown in Fig. 6(b).

$\Pr(\text{system fails}|\mathrm{PFE}_{1,2})$ can be computed as the sum of probabilities of all paths from root to sink node "1" in the generated

BDD:

$$\begin{aligned}\Pr(\text{system fails}|\text{PFE}_{1,2}) =\ & \Pr(S_{1C} \cap S_{2C}) \\ & + \Pr\left(S_{1C} \cap \bar{S}_{2C} \cap S_{3C} \cap S_{4C}\right) \\ & + \Pr\left(\bar{S}_{1C} \cap S_{3C} \cap S_{4C}\right). \end{aligned} \tag{17}$$

Similar to (1), we have

$$\begin{aligned}&\Pr(\text{system fails}|\text{TLFE}_1) \times \Pr(\text{TLFE}_1) \\ &= \Pr(\text{PFE}_{1,1}) + \Pr(\text{system fails}|\text{PFE}_{1,2}) \times \Pr(\text{PFE}_{1,2}). \end{aligned} \tag{18}$$

2) TLFE_2*:* r_1 fails locally and r_2 functions correctly. The occurrence probability of TLFE_2 is

$$\begin{aligned}\Pr(\text{TLFE}_2) &= \Pr\left(R_{1C} \cap \bar{R}_{2C}\right) \\ &= q_{r_1C}(t)\left[1 - q_{r_2C}(t)\right]. \end{aligned} \tag{19}$$

The LF of r_1 causes the probabilistic isolation effect to s_1. Since r_2 still functions, there is no failure isolation effect to s_2 or s_3. That is, $G(\text{TLFE}_2) = \{s_1\}$. For modeling the competition between LF of r_1 and PFGE of s_1, the following two isolation cases are identified.

a) $D_{2,1}$—s_1 is not isolated by LF of r_1.
b) $D_{2,2}$—s_1 is isolated by LF of r_1.

Then, we analyze the PFEs under each isolation case.

Under $D_{2,1}$, none of the biosensors is affected or isolated by corresponding relay LFs, i.e., $S(D_{2,1}) = NULL$. Thus, $\bar{S}(D_{2,1}) = \{s_1, s_2, s_3, s_4\}$. Two disjoint events can be distinguished under case $D_{2,1}$.

$D_{2,1} \cap \text{PFE}_{2,1}$*:* This event happens when at least one PFGE from biosensors in $S(D_{2,1})$ is not isolated, or any PFGE from biosensors in $\bar{S}(D_{2,1})$ takes place. Therefore, its occurrence probability is

$$\begin{aligned}\Pr(D_{2,1} \cap \text{PFE}_{2,1}) =\ & (1 - q_{R_1 \to S_1}) \times \Pr\left[\left(R_{1C} \cap \bar{R}_{2C}\right)\right. \\ & \left. \cap (S_{1P} \cup S_{2P} \cup S_{3P} \cup S_{4P})\right]. \end{aligned} \tag{20}$$

In the event of $D_{2,1} \cap \text{PFE}_{2,1}$ occurring, the entire WBAN system fails due to the failure propagation effect, and, thus, $\Pr(\text{system fails}|D_{2,1} \cap \text{PFE}_{2,1}) = 1$.

$D_{2,1} \cap \text{PFE}_{2,2}$: No failure propagation occurs to the WBAN system. Applying (9), the occurrence probability of $D_{2,1} \cap \text{PFE}_{2,2}$ is

$$\begin{aligned}\Pr(D_{2,1} \cap \text{PFE}_{2,2}) &= \Pr(\text{TLFE}_2) \times (1 - q_{R_1 \to S_1}) \\ &\quad - \Pr(D_{2,1} \cap \text{PFE}_{2,1}). \end{aligned} \tag{21}$$

In the case of $D_{2,1}$, the failed relay r_1 cannot cause failure isolation effect to the corresponding biosensor. Therefore, the reduced system FT model and BDD model given that event $D_{2,1} \cap \text{PFE}_{2,2}$ happens are the same as those in Fig. 6. Based on the BDD model of Fig. 6(b), we have

$$\begin{aligned}&\Pr(\text{system fails}|D_{2,1} \cap \text{PFE}_{2,2}) = \Pr(S_{1C} \cap S_{2C}) \\ &+ \Pr\left(S_{1C} \cap \bar{S}_{2C} \cap S_{3C} \cap S_{4C}\right) + \Pr\left(\bar{S}_{1C} \cap S_{3C} \cap S_{4C}\right). \end{aligned} \tag{22}$$

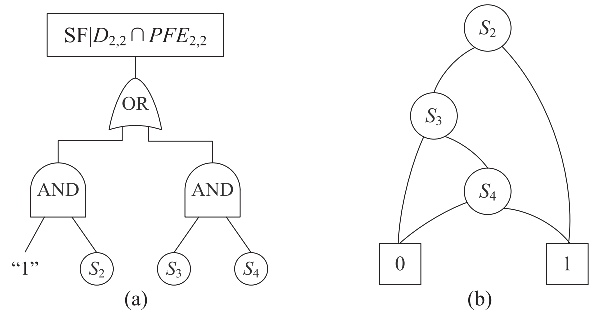

Fig. 7. Reduced system models for combined event $D_{2,2} \cap \text{PFE}_{2,2}$. (a) Reduced FT model. (b) BDD model.

Similarly, for the case $D_{2,2}$, $S(D_{2,2}) = \{s_1\}$ and $\bar{S}(D_{2,2}) = \{s_2, s_3, s_4\}$. The two disjoint events under case $D_{2,2}$ are as follows.

$D_{2,2} \cap \text{PFE}_{2,1}$: This event happens when any PFGE from biosensors in $\bar{S}(D_{2,2})$ occurs, or PFGE from biosensor in $S(D_{2,2})$ (i.e., s_1) happens before the LF of r_1. Let $X \to Y$ represent the event that *X* happens before *Y*. The occurrence probability of $D_{2,2} \cap \text{PFE}_{2,1}$ is

$$\begin{aligned}\Pr(D_{2,2} \cap \text{PFE}_{2,1}) &= q_{R_1 \to S_1} \times \Pr\{[(S_{2P} \cup S_{3P} \cup S_{4P}) \\ &\quad \cap R_{1C}] \cup (S_{1P} \to R_{1C})\} \times \Pr\left(\bar{R}_{2C}\right). \end{aligned} \tag{23}$$

In the event of $D_{2,2} \cap \text{PFE}_{2,1}$ occurring, the entire WBAN system fails. Thus, $\Pr(\text{system fails}|D_{2,2} \cap \text{PFE}_{2,1}) = 1$.

$D_{2,2} \cap \text{PFE}_{2,2}$: No failure propagation occurs to the WBAN system. According to (9), we have

$$\begin{aligned}\Pr(D_{2,2} \cap \text{PFE}_{2,2}) &= \Pr(\text{TLFE}_2) \times q_{R_1 \to S_1} \\ &\quad - \Pr(D_{2,2} \cap \text{PFE}_{2,1}). \end{aligned} \tag{24}$$

Note that this combined event covers the event that PFGE from s_1 happens after LF of r_1, and, thus, is isolated with occurrence probability $q_{R1 \to S1}$. Therefore, s_1 becomes inaccessible to the rest of the WBAN system. As shown in Fig. 7(a), the reduced system FT for the event $D_{2,2} \cap \text{PFE}_{2,2}$ can be generated by removing all relays and PFDG gates, and replacing failure event "S_1" with "1." Fig. 7(b) shows the corresponding BDD model for evaluating $\Pr\{\text{system fails}|D_{2,2} \cap \text{PFE}_{2,2}\}$.

Based on the BDD model in Fig. 7(b), we have

$$\begin{aligned}\Pr(\text{system fails}|D_{2,2} \cap \text{PFE}_{2,2}) =\ & \Pr(S_{2C}) \\ & + \Pr\left(\bar{S}_{2C} \cap S_{3C} \cap S_{4C}\right). \end{aligned} \tag{25}$$

According to (10), we integrate (20)–(25) and obtain

$$\begin{aligned}&\Pr(\text{system fails}|\text{TLFE}_2) \times \Pr(\text{TLFE}_2) \\ &= \sum_{k=1}^{2} \left[\Pr(D_{2,k} \cap \text{PFE}_{2,1}) + \Pr(\text{system fails}|D_{2,k} \cap \text{PFE}_{2,2})\right. \\ &\quad \left. \times \Pr(D_{2,k} \cap \text{PFE}_{2,2})\right]. \end{aligned} \tag{26}$$

3) TLFE_3: r_2 fails locally and r_1 functions correctly. The occurrence probability of TLFE_3 is

$$\begin{aligned}\Pr(\mathrm{TLFE}_3) &= \Pr\left(\bar{R}_{1C} \cap R_{2C}\right)\\ &= \left[1 - q_{r_1C}(t)\right] q_{r_2C}(t). \end{aligned} \tag{27}$$

The LF of r_2 causes probabilistic isolation effect to s_3. Since r_1 still functions, there is no isolation effect to s_1 or s_2. That is, $G(\mathrm{TLFE}_3) = \{s_3\}$. Two isolation cases are identified to handle the competition between LF of r_2 and PFGE of s_3 : $D_{3,1}$ − s_3 is not isolated by LF of r_2; $D_{3,2}$ − s_3 is isolated by LF of r_2.

Under $D_{3,1}$, $S(D_{3,1})$ is *NULL* since none of the biosensors is isolated by LF of r_2, and $\bar{S}(D_{3,1}) = \{s_1, s_2, s_3, s_4\}$. The two disjoint events under case $D_{3,1}$ are as follows.

$D_{3,1} \cap \mathrm{PFE}_{3,1}$: Since $S(D_{3,1})$ is *NULL*, this event happens when any PFGE from biosensors in $\bar{S}(D_{3,1})$ takes place. Therefore, the occurrence probability is

$$\begin{aligned}\Pr(D_{3,1} \cap \mathrm{PFE}_{3,1}) = &\ (1 - q_{R_2 \to S_3}) \times \Pr\left[\left(\bar{R}_{1C} \cap R_{2C}\right)\right.\\ &\left. \cap (S_{1P} \cup S_{2P} \cup S_{3P} \cup S_{4P})\right]. \end{aligned} \tag{28}$$

In the event of $D_{3,1} \cap \mathrm{PFE}_{3,1}$ occurring, the failure propagation effect causes the entire WBAN system failure, we have $\Pr(\text{system fails}|D_{3,1} \cap PFE_{3,1}) = 1$.

$D_{3,1} \cap \mathrm{PFE}_{3,2}$: No failure propagation occurs to the WBAN system. By applying (9), the occurrence probability of $D_{3,1} \cap \mathrm{PFE}_{3,2}$ is

$$\begin{aligned}\Pr(D_{3,1} \cap \mathrm{PFE}_{3,2}) = &\ \Pr(\mathrm{TLFE}_3) \times (1 - q_{R_2 \to S_3})\\ &- \Pr(D_{3,1} \cap \mathrm{PFE}_{3,1}). \end{aligned} \tag{29}$$

In the case of $D_{3,1}$, the failed relay r_2 cannot cause isolation effect to the corresponding biosensor. The reduced system models given that event $D_{3,1} \cap \mathrm{PFE}_{3,2}$ happens are the same as those in Fig. 6. Therefore, we have

$$\begin{aligned}&\Pr(\text{system fails}|D_{3,1} \cap \mathrm{PFE}_{3,2}) = \Pr(S_{1C} \cap S_{2C})\\ &+ \Pr\left(S_{1C} \cap \bar{S}_{2C} \cap S_{3C} \cap S_{4C}\right) + \Pr\left(\bar{S}_{1C} \cap S_{3C} \cap S_{4C}\right). \end{aligned} \tag{30}$$

Similarly, for the case $D_{3,2}$, $S(D_{3,2}) = \{s_3\}$ and $\bar{S}(D_{3,2}) = \{s_1, s_2, s_4\}$. The two disjoint events under case $D_{3,2}$ are as follows.

$D_{3,2} \cap \mathrm{PFE}_{3,1}$: This event happens when any PFGE from biosensors in $\bar{S}(D_{3,2})$ occurs, or PFGE from biosensor in $S(D_{3,2})$ (i.e., s_3) happens before the LF of r_2. The occurrence probability is

$$\begin{aligned}\Pr(D_{3,2} \cap \mathrm{PFE}_{3,1}) = &\ q_{R_2 \to S_3} \times \Pr\{[(S_{1P} \cup S_{2P} \cup S_{4P})\\ &\cap R_{2C}] \cup (S_{3P} \to R_{2C})\} \times \Pr(\bar{R}_{1C}). \end{aligned} \tag{31}$$

$D_{3,2} \cap \mathrm{PFE}_{3,2}$: No failure propagation occurs to the WBAN system. The occurrence probability is computed by applying (9) as

$$\begin{aligned}\Pr(D_{3,2} \cap \mathrm{PFE}_{3,2}) = &\ \Pr(\mathrm{TLFE}_3) \times q_{R_2 \to S_3}\\ &- \Pr(D_{3,2} \cap \mathrm{PFE}_{3,1}). \end{aligned} \tag{32}$$

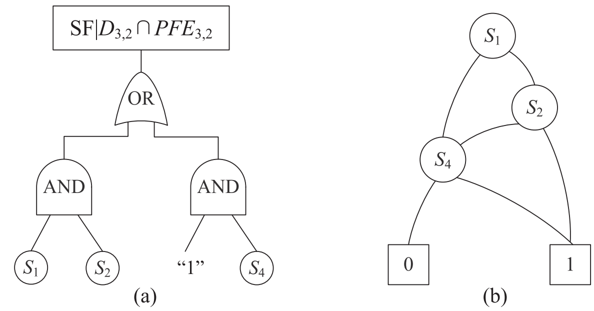

Fig. 8. Reduced system models for combined event $D_{3,2} \cap \mathrm{PFE}_{3,2}$. (a) Reduced FT model. (b) BDD model.

TABLE III
ISOLATION CASES FOR TLFE_4

Case #	Case definition	Occurrence probability $\Pr(D_{4,k})$
$D_{4,1}$	$\bar{S}_1 \cap \bar{S}_2 \cap \bar{S}_3$	$(1 - q_{R1\to S1})(1 - q_{R1\cap R2\to S2})(1 - q_{R2\to S3})$
$D_{4,2}$	$S_1 \cap \bar{S}_2 \cap \bar{S}_3$	$(q_{R1\to S1})(1 - q_{R1\cap R2\to S2})(1 - q_{R2\to S3})$
$D_{4,3}$	$\bar{S}_1 \cap S_2 \cap \bar{S}_3$	$(1 - q_{R1\to S1})(q_{R1\cap R2\to S2})(1 - q_{R2\to S3})$
$D_{4,4}$	$\bar{S}_1 \cap \bar{S}_2 \cap S_3$	$(q_{R2\to S3})(1 - q_{R1\to S1})(1 - q_{R1\cap R2\to S2})$
$D_{4,5}$	$S_1 \cap S_2 \cap \bar{S}_3$	$(q_{R1\to S1})(q_{R1\cap R2\to S2})(1 - q_{R2\to S3})$
$D_{4,6}$	$S_1 \cap \bar{S}_2 \cap S_3$	$(q_{R1\to S1})(1 - q_{R1\cap R2\to S2})(q_{R2\to S3})$
$D_{4,7}$	$\bar{S}_1 \cap S_2 \cap S_3$	$(1 - q_{R1\to S1})(q_{R1\cap R2\to S2})(q_{R2\to S3})$
$D_{4,8}$	$S_1 \cap S_2 \cap S_3$	$(q_{R1\to S1})(q_{R1\cap R2\to S2})(q_{R2\to S3})$

TABLE IV
AFFECTED BIOSENSORS ILLUSTRATION FOR TLFE_4

Case #	Case definition	$S(D_{4,k})$	$\bar{S}(D_{4,k})$
$D_{4,1}$	$\bar{S}_1 \cap \bar{S}_2 \cap \bar{S}_3$	*NULL*	$\{s_1, s_2, s_3, s_4\}$
$D_{4,2}$	$S_1 \cap \bar{S}_2 \cap \bar{S}_3$	$\{s_1\}$	$\{s_2, s_3, s_4\}$
$D_{4,3}$	$\bar{S}_1 \cap S_2 \cap \bar{S}_3$	$\{s_2\}$	$\{s_1, s_3, s_4\}$
$D_{4,4}$	$\bar{S}_1 \cap \bar{S}_2 \cap S_3$	$\{s_3\}$	s_1, s_2, s_4
$D_{4,5}$	$S_1 \cap S_2 \cap \bar{S}_3$	$\{s_1, s_2\}$	$\{s_3, s_4\}$
$D_{4,6}$	$S_1 \cap \bar{S}_2 \cap S_3$	$\{s_1, s_3\}$	$\{s_2, s_4\}$
$D_{4,7}$	$\bar{S}_1 \cap S_2 \cap S_3$	$\{s_2, s_3\}$	$\{s_1, s_4\}$
$D_{4,8}$	$S_1 \cap S_2 \cap S_3$	$\{s_1, s_2, s_3\}$	$\{s_4\}$

The $D_{3,2} \cap \mathrm{PFE}_{3,2}$ covers the event that PFGE from s_3 happens after LF of r_2 and is isolated with occurrence probability $q_{R2\to S3}$.That is, s_3 becomes inaccessible to the rest of the system. The reduced system models are presented in Fig. 8.

Based on the BDD model in Fig. 8(b), we have

$$\begin{aligned}\Pr(\text{system fails}|D_{3,2} \cap \mathrm{PFE}_{3,2}) = &\ \Pr(S_{1C} \cap S_{2C})\\ &+ \Pr\left(S_{1C} \cap \bar{S}_{2C} \cap S_{4C}\right)\\ &+ \Pr\left(\bar{S}_{1C} \cap S_{4C}\right). \end{aligned} \tag{33}$$

Therefore, according to (10), (28)—(33) are integrated to obtain

$$\begin{aligned}&\Pr(\text{system fails}|\mathrm{TLFE}_3) \times \Pr(\mathrm{TLFE}_3)\\ &= \sum_{k=1}^{2} [\Pr(D_{3,k} \cap \mathrm{PFE}_{3,1}) + \Pr(\text{system fails}|D_{3,k} \cap \mathrm{PFE}_{3,2})\\ &\quad \times \Pr(D_{3,k} \cap \mathrm{PFE}_{3,2})]. \end{aligned} \tag{34}$$

TABLE V
OCCURRENCE PROBABILITY OF EVENTS $D_{4,k} \cap \mathrm{PFE}_{4,1}$

Case #	$\Pr(D_{4,k} \cap \mathrm{PFE}_{4,1})$
$D_{4,1}$	$\Pr(D_{4,1}) \times \Pr[(R_{1C} \cap R_{2C}) \cap (S_{1P} \cup S_{2P} \cup S_{3P} \cup S_{4P})]$
$D_{4,2}$	$\Pr(D_{4,2}) \times \Pr\{[R_{1C} \cap (S_{2P} \cup S_{3P} \cup S_{4P})] \cup (S_{1P} \to R_{1C})\} \times \Pr(R_{2C})$
$D_{4,3}$	$\Pr(D_{4,3}) \times \Pr\left\{\begin{matrix}[(R_{1C} \cap R_{2C}) \cap (S_{1P} \cup S_{3P} \cup S_{4P})] \cup \\ [(S_{2P} \to R_{1C}) \cap R_{2C}] \cup [(S_{2P} \to R_{2C}) \cap R_{1C}]\end{matrix}\right\}$
$D_{4,4}$	$\Pr(D_{4,4}) \times \Pr\{[R_{2C} \cap (S_{1P} \cup S_{2P} \cup S_{4P})] \cup (S_{3P} \to R_{2C})\} \times \Pr(R_{1C})$
$D_{4,5}$	$\Pr(D_{4,5}) \times \Pr\left\{\begin{matrix}[(R_{1C} \cap R_{2C}) \cap (S_{3P} \cup S_{4P})] \cup [(S_{1P} \to R_{1C}) \cap R_{2C}] \\ \cup [(S_{2P} \to R_{1C}) \cap R_{2C}] \cup [(S_{2P} \to R_{2C}) \cap R_{1C}]\end{matrix}\right\}$
$D_{4,6}$	$\Pr(D_{4,6}) \times \Pr\left\{\begin{matrix}[(R_{1C} \cap R_{2C}) \cap (S_{2P} \cup S_{4P})] \cup \\ [(S_{1P} \to R_{1C}) \cap R_{2C}] \cup [(S_{3P} \to R_{2C}) \cap R_{1C}]\end{matrix}\right\}$
$D_{4,7}$	$\Pr(D_{4,7}) \times \Pr\left\{\begin{matrix}[(R_{1C} \cap R_{2C}) \cap (S_{1P} \cup S_{4P})] \cup [(S_{2P} \to R_{1C}) \cap R_{2C}] \\ \cup [(S_{2P} \to R_{2C}) \cap R_{1C}] \cup [(S_{3P} \to R_{2C}) \cap R_{1C}]\end{matrix}\right\}$
$D_{4,8}$	$\Pr(D_{4,8}) \times \Pr\left\{\begin{matrix}[R_{1C} \cap R_{2C} \cap S_{4P}] \cup [(S_{1P} \to R_{1C}) \cap R_{2C}] \\ \cup [(S_{3P} \to R_{2C}) \cap R_{1C}] \cup [(S_{2P} \to R_{1C}) \cap R_{2C}] \\ \cup [(S_{2P} \to R_{2C}) \cap R_{1C}]\end{matrix}\right\}$

4) TLFE_4: Both r_1 and r_2 fail locally and cause probabilistic isolation effects to s_1, s_2, and s_3 with conditional probabilities $q_{R1\to S1}$, $q_{R1\cap R2\to S2}$, and $q_{R2\to S3}$, respectively. The occurrence probability of TLFE_4 is

$$\Pr(\mathrm{TLFE}_4) = \Pr(R_{1C} \cap R_{2C}) = q_{r_1C}(t)\, q_{r_2C}(t). \tag{35}$$

As illustrated in Table II, $G(\mathrm{TLFE}_4) = \{s_1, s_2, s_3\}$, three biosensors are associated with the competitions in this event. Therefore, $2^3 = 8$ isolation cases are distinguished as shown in Table III. In particular, each isolation case is a combination of events representing biosensors are isolated or not isolated by their corresponding relay LFs.

For each case $D_{4,k}$, the set of biosensors affected by their corresponding relay LFs (referred to as $S(D_{4,k})$) and the set of biosensors which are not involved in the competition (referred to as $\bar{S}(D_{4,k})$) are listed in the Table IV. Under each case $D_{4,k}$, the two disjoint combined events are as follows.

$D_{4,k} \cap \mathrm{PFE}_{4,1}$: The failure propagation effect occurs. That is, at least one PFGE from biosensors in $S(D_{4,k})$ is not isolated, or any PFGE from biosensors in $\bar{S}(D_{4,k})$ takes place.

$D_{4,k} \cap \mathrm{PFE}_{4,2}$: No failure propagation occurs to the system. This happens when all PFGEs from biosensors in $S(D_{4,k})$ are isolated and no PFGE from biosensors in $\bar{S}(D_{4,k})$ occurs, or no PFGE occurs to any biosensors.

Similar to the procedure used for analyzing TLFE_2 and TLFE_3, the occurrence probabilities of events $D_{4,k} \cap \mathrm{PFE}_{4,1}$ are evaluated for all the eight isolation cases as shown in Table V.

As defined earlier, the notation "$X \to Y$" is used to represent the sequential event that X happens before Y. For example, the term $[(S_{2P} \to R_{1C}) \cap R_{2C}]$ in Table V represents that both r_1 and r_2 fail locally, and PFGE from s_2 happens before r_1 fails locally. Similarly, the term $[(S_{2P} \to R_{2C}) \cap R_{1C}]$ represents that both r_1 and r_2 fail locally, and PFGE from s_2 happens before r_2 fails locally. As an example, we illustrate the evaluation of $\Pr[(S_{2P} \to R_{1C}) \cap R_{2C}]$ below.

Since the two events $S_{2P} \to R_{1C}$ and R_{2C} are independent, we have $\Pr[(S_{2P} \to R_{1C}) \cap R_{2C}] = Pr(S_{2P} \to R_{1C}) \times \Pr(R_{2C})$, where the occurrence probability of the sequential event (i.e., $S_{2P} \to R_{1C}$) can be calculated using double integral [2] as

$$\Pr(S_{2P} \to R_{1C}) = \int_0^t \int_{\tau_1}^t f_{s_2P}(\tau_1) f_{r_1C}(\tau_2)\, d\tau_2 d\tau_1,$$

where

$$f_{r_1C}(t) = {}^{d}\!\left(\frac{q_{r_1L}(t)}{1 - q_{r_1P}(t)}\right)\Big/_{\mathrm{dt}}. \tag{36}$$

$f_{s2P}(t)$ in (36) is the pdf of time-to-PFGE of biosensor s_2. In the case of time-to-LFs and PFGEs of components, following the Weibull distribution with shape parameter (α_i), scale parameter (β_i), and pdf of $f_i(t) = \alpha_i \beta_i^{\alpha_i} t^{\alpha_i - 1} e^{-(\beta_i t)^{\alpha_i}}$, the evaluation of $\Pr[(S_{2P} \to R_{1C}) \cap R_{2C}]$ can be elaborated as

$$\begin{aligned}
&\Pr[(S_{2P} \to R_{1C}) \cap R_{2C}] \\
&= \Pr(R_{2C}) \times \Pr(S_{2P} \to R_{1C}) \\
&= \left\{ e^{(\beta_{r_2P} t)^{\alpha_{r_2P}}} - e^{\left[(\beta_{r_2P} t)^{\alpha_{r_2P}} - (\beta_{r_2L} t)^{\alpha_{r_2L}}\right]} \right\} \\
&\times \int_0^t \int_{\tau_1}^t \left\{ \begin{matrix} \alpha_{s_2P} {\beta_{s_2P}}^{\alpha_{s_2P}} {\tau_1}^{\alpha_{s_2P} - 1} e^{-(\beta_{s_2P} \tau_1)^{\alpha_{s_2P}}} \times \\ \left\{ \begin{matrix} \alpha_{r_1P} {\beta_{r_1P}}^{\alpha_{r_1P}} {\tau_2}^{\alpha_{r_1P} - 1} e^{(\beta_{r_1P} \tau_2)^{\alpha_{r_1P}}} - \\ \left(\begin{matrix} \alpha_{r_1P} {\beta_{r_1P}}^{\alpha_{r_1P}} {\tau_2}^{\alpha_{r_1P} - 1} \\ - \alpha_{r_1L} {\beta_{r_1L}}^{\alpha_{r_1L}} {\tau_2}^{\alpha_{r_1L} - 1} \end{matrix} \right) \times \\ e^{\left[(\beta_{r_1P} \tau_2)^{\alpha_{r_1P}} - (\beta_{r_1L} \tau_2)^{\alpha_{r_1L}}\right]} \end{matrix} \right\} \end{matrix} \right\} d\tau_2 d\tau_1.
\end{aligned} \tag{37}$$

As stated in Section V–A, all the Weibull distribution parameters associated with biosensors and relays are known input, and, thus, are constant in (37). The double integral in (37) can be computed using Mathcad tool, which calculates multiple integrals numerically using the Romberg algorithm [38].

Based on the evaluation results from Tables III and V, $\Pr(D_{4,k} \cap \mathrm{PFE}_{4,2})$ can be obtained for all eight isolation cases by applying (9) as

$$\Pr\left(D_{4,k} \cap \mathrm{PFE}_{4,2}\right) = \Pr\left(\mathrm{TLFE}_4\right) \times \Pr\left(D_{4,k}\right) - \Pr\left(D_{4,k} \cap \mathrm{PFE}_{4,1}\right). \tag{38}$$

For evaluating the conditional system unreliability $\Pr(\text{system fails}|D_{4,k} \cap \mathrm{PFE}_{4,2})$, the reduced system models under events $D_{4,1} \cap \mathrm{PFE}_{4,2}$, $D_{4,2} \cap \mathrm{PFE}_{4,2}$, and $D_{4,4} \cap \mathrm{PFE}_{4,2}$ are the same as the models in Figs. 6–8, correspondingly. Fig. 9 presents the reduced system FT models and corresponding BDD models for the other combined events. Note that the FT models in Fig. 9(d) and (e) can be simply evaluated as constant "1" after Boolean reduction, so their BDDs are not shown.

The evaluation results of $\Pr(\text{system fails}|D_{4,k} \cap \mathrm{PFE}_{4,2})$ are summarized in Table VI.

Integrating the results from Tables V and VI and (38), we have

$$\Pr\left(\text{system fails}|\mathrm{TLFE}_4\right) \times \Pr\left(\mathrm{TLFE}_4\right) = \sum_{k=1}^{8}\left[\Pr\left(D_{4,k} \cap \mathrm{PFE}_{4,1}\right) + \Pr\left(\text{system fails}|D_{4,k} \cap \mathrm{PFE}_{4,2}\right) \times \Pr\left(D_{4,k} \cap \mathrm{PFE}_{4,2}\right)\right]. \tag{39}$$

Step 4: Integration for System Unreliability

Based on the results obtained from Step 3, $\Pr(\text{system fails}|\mathrm{TPFE}_2)$ can be computed by integrating (18), (26), (34), and (39) into (13). Finally, the unreliability of the example WBAN system is evaluated using (2).

C. Evaluation Results and Discussion

To illustrate the flexibility in distribution types of the proposed method, we calculate unreliability of the example WBAN system using different distributions for both component failures and probabilistic isolation factors. Table VII lists component failure parameters, where time-to-LFs and PFGEs of components follows the Weibull distribution with shape parameter (α) and scale parameter (β). Note that the Weibull distribution is reduced to an exponential distribution when α is 1. Table VIII lists different probabilistic isolation factor groups (IFGs) allocated for the three PFD groups. Particularly, in IFG_7, the probabilistic isolation factors follow Weibull distributions with shape (α) and scale (β) parameters, and *cdf* in the form of $F(t) = 1 - e^{-(\beta t)^{\alpha}}$. IFG_7 demonstrates that the energy of a battery powered biosensor decreases as time proceeds and becomes inadequate for a direct transmission to the sink device at a certain time instant. Specifically, following the specified Weibull distribution, as time proceeds, the remaining energy decreases leading to a larger probability of the biosensor being isolated, thus the probabilistic isolation factor increases. At a certain mission time *t*, the probabilistic isolation factor is fixed representing the probability that the remaining energy is insufficient for a direct transmission to the sink device, i.e., the biosensor is isolated.

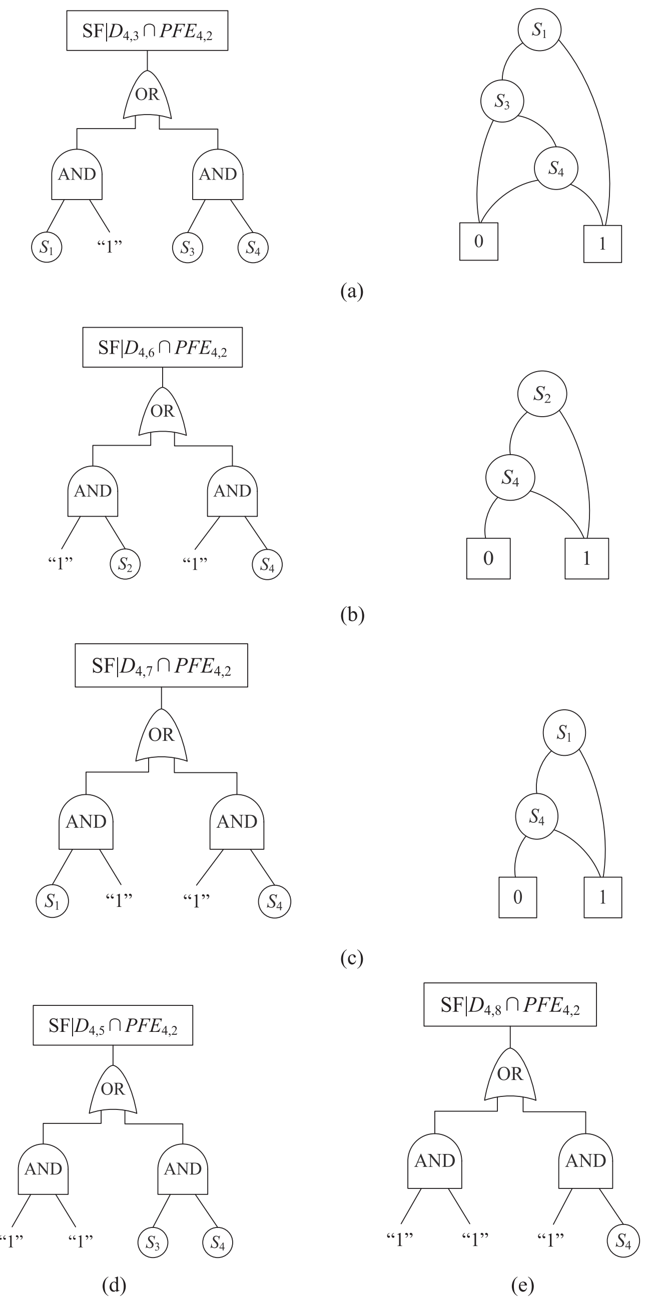

Fig. 9. Reduced system models for events $D_{4,k} \cap \mathrm{PFE}_{4,2}$. (a) Reduced FT and BDD model for $D_{4,3} \cap \mathrm{PFE}_{4,2}$. (b) Reduced FT and BDD model for $D_{4,6} \cap \mathrm{PFE}_{4,2}$. (c) Reduced FT and BDD model for $D_{4,7} \cap \mathrm{PFE}_{4,2}$. (d) Reduced FT for $D_{4,5} \cap \mathrm{PFE}_{4,2}$. (e) Reduced FT model for $D_{4,8} \cap \mathrm{PFE}_{4,2}$.

Unreliabilities of the example WBAN system at several different mission times are presented in Table IX. The comparative system unreliabilities under different IFGs are illustrated in Figs. 10 and 11.

The proposed procedure addresses special system models by using different IFGs such as those in Table VIII. For instance, with IFG_1, we actually have a system without any PFD

TABLE VI
CONDITIONAL SYSTEM FAILURE PROBABILITY UNDER EVENTS $D_{4,k} \cap \mathrm{PFE}_{4,2}$

Case #	$\Pr(\text{system fails}\mid D_{4,k} \cap \mathrm{PFE}_{4,2})$
$D_{4,1}$	$\Pr(S_{1C} \cap S_{2C}) + \Pr(S_{1C} \cap \bar{S}_{2C} \cap S_{3C} \cap S_{4C}) + \Pr(\bar{S}_{1C} \cap S_{3C} \cap S_{4C})$
$D_{4,2}$	$\Pr(S_{2C}) + \Pr(\bar{S}_{2C} \cap S_{3C} \cap S_{4C})$
$D_{4,3}$	$\Pr(S_{1C}) + \Pr(\bar{S}_{1C} \cap S_{3C} \cap S_{4C})$
$D_{4,4}$	$\Pr(S_{1C} \cap S_{2C}) + \Pr(S_{1C} \cap \bar{S}_{2C} \cap S_{4C}) + \Pr(\bar{S}_{1C} \cap S_{4C})$
$D_{4,5}$	1
$D_{4,6}$	$\Pr(S_{2C}) + \Pr(\bar{S}_{2C} \cap S_{4C})$
$D_{4,7}$	$\Pr(S_{1C}) + \Pr(\bar{S}_{1C} \cap S_{4C})$
$D_{4,8}$	1

TABLE VII
WBAN COMPONENT FAILURE PARAMETERS (RATES ARE IN PER HOUR)

Component	LF	PF
s_i	$\beta = 2\mathrm{e}-4, \alpha = 2$	$\beta = 1\mathrm{e}-5, \alpha = 2$
r_i	$\beta = 1\mathrm{e}-4, \alpha = 1$	$\beta = 1\mathrm{e}-5, \alpha = 1$

TABLE VIII
PROBABILISTIC IFGS

IFG #	$q_{R1\to S1}$	$q_{R1\cap R2\to S2}$	$q_{R2\to S3}$
IFG_1	0	0	0
IFG_2	1	0	0
IFG_3	0	1	0
IFG_4	1	0	1
IFG_5	1	1	0
IFG_6	1	1	1
IFG_7	$\beta = 2\mathrm{e}-4, \alpha = 1$	$\beta = 2.5\mathrm{e}-4, \alpha = 1$	$\beta = 2.2\mathrm{e}-4, \alpha = 1$

TABLE IX
SYSTEM UNRELIABILITIES AT SELECTED MISSION TIMES (HOURS)

IFG	$t = 1000$	$t = 2000$	$t = 3000$	$t = 4000$	$t = 5000$
IFG_1	0.023204	0.082262	0.225597	0.447908	0.679195
IFG_2	0.026741	0.104091	0.273544	0.508638	0.730526
IFG_3	0.023545	0.086303	0.238414	0.468768	0.700454
IFG_4	0.030265	0.125401	0.318523	0.562688	0.773644
IFG_5	0.035426	0.131441	0.315976	0.552804	0.764184
IFG_6	0.038623	0.149233	0.351118	0.592695	0.794282
IFG_7	0.024950	0.101570	0.282015	0.532041	0.757826

behavior; with IFG_2 and IFG_3, the DFD behavior involving one PFD group is considered; with IFG_4, the DFD behavior involving two independent PFD groups is considered; with IFG_5 and IFG_6, the DFD behavior involving two or three correlated PFD groups is considered; with IFG_7, the PFD behavior involving three correlated PFD groups is considered. The parameter setting of IFG_7 demonstrates that the proposed method is applicable to arbitrary types of failure isolation distributions.

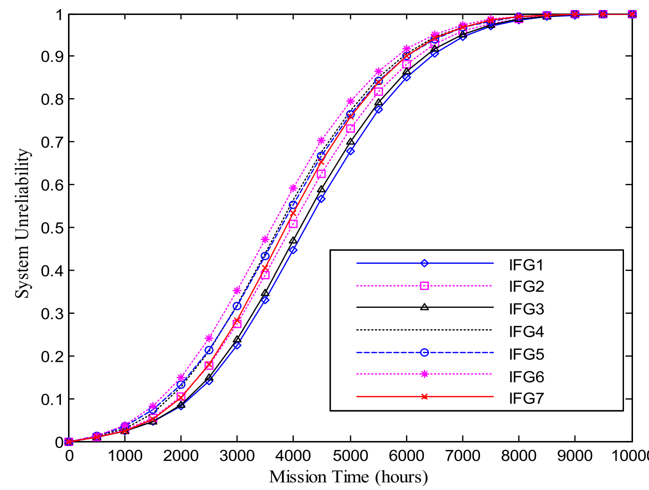

Fig. 10. Unreliabilities of the example WBAN for mission time from 0 to 10000 h.

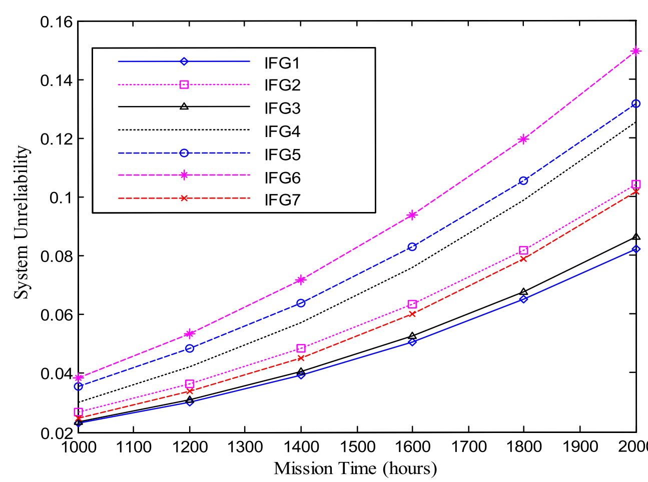

Fig. 11. Unreliabilities of the example WBAN (zoomed results for mission time from 1000 to 2000 h).

It is worth stating that for systems subject to correlated probabilistic competing failures, the failure isolation causes both reliability improvement effect (i.e., PFGEs from isolated components are prevented from affecting the rest of system) and deterioration effect (i.e., the isolated components are disabled and regarded as being locally failed); which one of the two effects dominating is determined by the specific parameter settings.

Based on the evaluation results in Table IX and Figs. 10 and 11, the unreliability of the example WBAN under IFG_1 is the lowest (no failure isolation exists), while the system unreliability under IFG_6 is the highest (three biosensors are isolated by the relay LFs). This implies that the deterioration effect of failure isolation dominates under the parameter setting used for the example WBAN system. The deterioration effect increases as the number of biosensors involved in the DFD behavior increases, which is shown by results of IFG_1 to IFG_6.

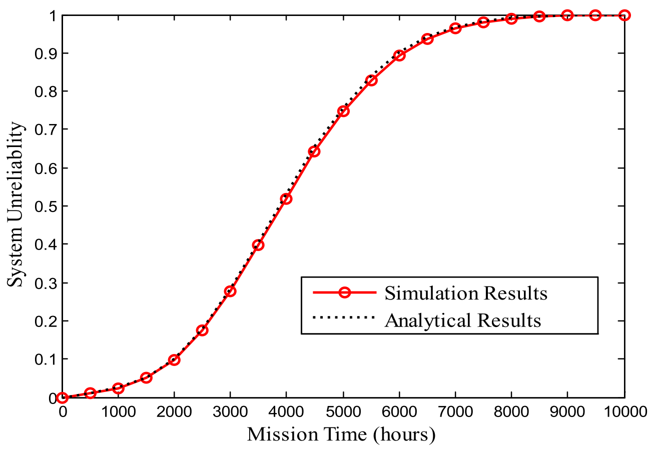

Fig. 12. Comparative results of WBAN unreliablities.

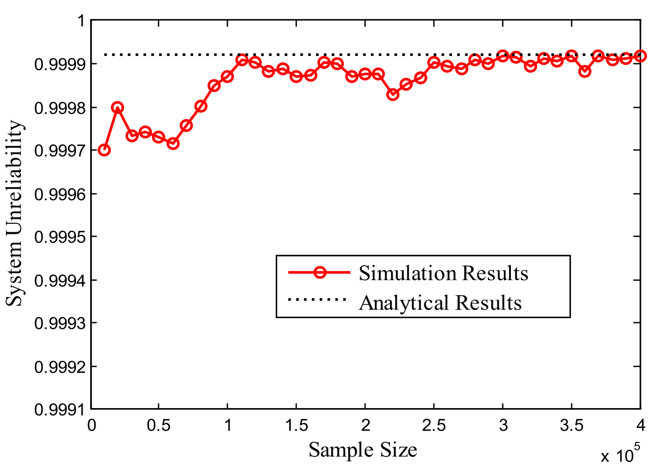

Fig. 13. Effects of sample size used in simulations.

For systems with only one biosensor being subject to DFD, the unreliability under IFG_3 (the biosensor is isolated by combined relay LF events) is lower than that under IFG_2 (the biosensor is isolated by LF of a single relay) since the probability that the failure isolation occurs is smaller under IFG_3.

D. Verification of the Proposed Method

To verify the correctness of the proposed method, Monte–Carlo simulation is conducted for analyzing the same example WBAN system using the component failure parameters in Table VII and probabilistic isolation factors of IFG_7 in Table VIII. The simulations are performed in Python 2.7.10 running on a laptop with Intel Core i7-2670QM CPU at 2.20-GHz and 8-GB RAM.

Fig. 12 shows unreliabilities of the example WBAN at different mission times (from 0 to 10000 h) obtained using simulations with 10^5 sample size and using the proposed method. The simulation results are consistent with the analytical results obtained using the proposed method. Sample size affects accuracy of simulation results. In general, simulation results are more accurate as the sample size increases. Fig. 13 illustrates the simulation results for $t = 10000$ h are closer to the analytical results in the general trend as the sample size increases from 10^4 to 4×10^5.

Note that the Markov-model-based method is another potential method for handling the complicated PFD behavior considered in this study. However, it is typically limited to exponential component failure distributions and isolation factors. Semi-Markov processes can handle nonexponential distributions [39]. However, they share the same state-space explosion problem as in the Markov methods, which make solution to the generated models computationally intensive and intractable.

VI. Conclusion and Future Work

The correlated PFD behavior exists in many real-world systems, such as body area networks, wireless sensor networks, and computer systems. The twofold effects, i.e., failure isolation and failure propagation effects as well as the competitions between these effects must be considered in system modeling and analysis. In this paper, we make new contributions by proposing a combinatorial and analytical method to analyze reliability of nonrepairable binary systems subject to the correlated PFD behaviors. As illustrated through the detailed analysis of an example WBAN system, the proposed method is applicable to arbitrary types of distributions for system component failure times and probabilistic failure isolation factors. Correctness of the proposed method is verified using Monte–Carlo simulations.

In the future work, we will be interested in exploring experimental verification of the proposed analytical method using real WBAN systems. We will also investigate and extend the algebraic method proposed in [40] to model systems subject to probabilistic competition effects and complex *s*-relationships among component failure events. This paper assumes that all system components have binary states (operation or failure) and are involved in a single-phased mission. As another direction of our future work, the proposed methodology will be extended to address reliability of multistate systems and phased-mission systems.

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